Some Optimization Models for Listening Room Design

Marcelo Queiroz

Institute of Mathematics and Statistics – University of São Paulo (USP)
Rua do Matão 1010, Cidade Universitária, 05508-090 São Paulo, SP, Brazil

`mqz@ime.usp.br`

Abstract

In this paper some preliminary formulations for the simulation and design of critical listening rooms using optimization techniques aiming at nearly flat frequency response are discussed. Emphasis is given to the computation of an approximation of frequency response as a function of the design variables, i.e. without directly measuring this response in a real room or a scale model. Optimization models for room design are given, which take into account a range of listening positions (e.g. an audience) and sound sources (e.g. a stage). Some implementation issues are pointed out and further research directions are laid out.

1 Introduction

The design of critical listening rooms (e.g. project studios, small concert halls, home theaters) is a problem which poses important questions before it can be defined: in the first place the question of what exactly is to be designed; secondly, what is it at which the design aims.

To simplify discussion a cuboid room is assumed, though much of the material may be adapted to polyhedral rooms (described by a finite number of planes). The main design variables considered here are the room dimensions and reflection coefficients of the surfaces (walls, floor, ceiling), where ranges of possible room dimensions, reflection coefficients and positions of sound sources and listeners are given. It is assumed that sound is originated in any of the possible sound source positions (as in a stage) and may be received/perceived in any of the listening positions (as in an audience): no optimal placement of listeners or sound sources is sought after.

The main goal is to obtain a nearly flat frequency response over the range of possible source and listener positions, where "nearly flat" presupposes the minimization of some measure of deviation from flatness, such as the standard deviation or least-squares errors of the frequency response averaged over source-listener pairs, or the supremum of such values over source-listener pairs. The preference of least-squares error over standard deviation is indicative of the fact that an affine frequency response can be lossless electronically equalized to a flat (horizontal) response and so is practically as good as a flat response (at least in the studio setting).

There are two major concerns in this project. The first one is to have robust models and prediction tools that may be used in simulation without need of directly measuring acoustic data in a real room or a scale model. This is clearly a *sine qua non* condition for the automatic search for an optimal project. The other concern is to have open formulations and algorithms, in the sense of providing detailed descriptions of the optimization models and methods, as in [Iazzetta, Kon & Silva 2001]. An implicit risk is taken of repeating methods and formulations that may be found elsewhere (e.g. [Rindel 1997, Rindel 2000, D'Antonio & Cox 1997, Warusfel 1995]), but not explicitly enough to be readily implemented.

The next section analyses the computation of an approximation of frequency response as a function of the design variables, taking into account such effects as source-boundary interference (in particular modal coupling and comb filtering) and other amplification/damping effects due to first reflections. Section 3 treats the optimization models and available methods for solving them. Other possible design variables and quality criteria are later discussed as further research topics. As stated in the first line of the abstract this is a preliminary work, and so much of the description of methods and required computational tests are not yet finished at the time of writing.

2 Approximating Frequency Response

In this section some of the main phenomena that imply amplification or damping of specific frequencies in a room are considered. For each type of phenomenon the special case of specular reflection is considered first, and some discussion on the effect of diffusion follows. See [Beranek 1962, Beranek 1993] for more information on the general acoustical problem of reflection and diffusion in rooms.

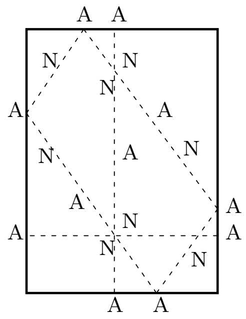
2.1 Modal Coupling

Modal coupling is a result of the interaction of the sound source and the reflective surfaces of a room. It is associated with closed specular-reflection paths (loops) whose lengths are multiples of the wavelength considered. It affects mainly those listening positions that lie within the closed paths, though diffusion may blur this effect to adjacent positions.

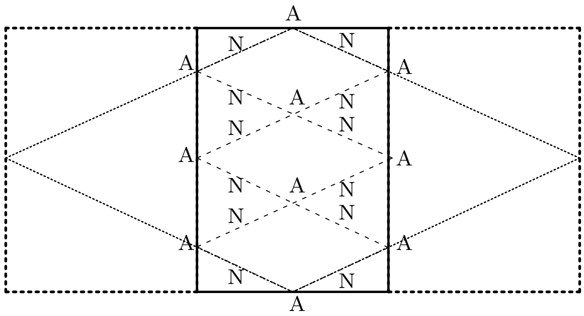
According to how many surfaces the corresponding closed path hits, room modes are called axial, tangential and oblique. These are related with having one, two or three non-zero values of n in the formula $f(n_x, n_y, n_z) = \frac{c}{2}\sqrt{\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2}$, where f is the frequency, c is the speed of sound and the room dimensions are L_x, L_y and L_z.

Axial modes are the easiest to calculate and to locate its corresponding nodes (where vibration is damped). Considering $n_y = n_z = 0$ in the above formula, axial modes parallel to the x dimension have frequencies given by $f_n = \frac{cn}{2L_x}$ and nodes located in the positions $d(n) = \left\{\frac{kL_x}{2n} \mid k \text{ odd},\ k < 2n\right\}$. Assuming the reflection coefficients of both surfaces involved to be the same and equal to $\varrho \in (0,1)$ for a given frequency $f(n)$, the amplification factor at the antinodes, which occur at the positions $a(n) = \left\{\frac{kL_x}{2n} \mid k \text{ even},\ k \leq 2n\right\}$, can be computed as $1 + \varrho + \varrho^2 + \cdots = \frac{1}{1-\varrho}$, whereas the damping factor at the nodes can be computed as $1 - \varrho + \varrho^2 - \varrho^3 + \cdots = \frac{1-\varrho}{(1-\varrho^2)(1-\varrho^3)}$.

Tangential modes involve four reflecting surfaces in the room. The simplest ones correspond to simple parallelograms hitting each surface once. These can be easily shown to have their sides parallel to each diagonal of a square orthogonal to the surfaces involved. More complicate paths correspond to folded parallelograms of a virtual room consisting of specular copies of the original room, as indicated in the sketch below. The letter N and A denote nodes and antinodes, respectively.



A simple tangential mode with two axial modes



A tangential mode corresponding to a folded paralellogram

It can be easily checked that closed paths corresponding to oblique modes can be obtained by starting at the sound source and following the direction $(n_x L_x, n_y L_y, n_z L_z)$, where n_x, n_y, n_z are natural numbers.

It is assumed that the sound source is located in the paths, so that in the first sketch the source is at the intersection of the three modes. The amplifying and damping factor for these modes can be easily calculated by using the appropriate reflection coefficients; the tangential mode in the first sketch for instance would have an amplification of $\frac{1}{1-\varrho_1\varrho_2\varrho_3\varrho_4}$ at the antinodes. Clearly the more surfaces the closed path hits, the smaller the influence of that particular room mode in amplification and damping.

It is important to note that the above examples are oversimplifications for the case of pure specular reflections. Due to diffusion, modal coupling effects will be perceived in regions containing these nodes and antinodes, the intensity of the effect lessening with the distance to the geometrical location of the node or antinode. Also notice that all reflection coefficients are frequency-related, usually decreasing as frequency increases.

With that in mind, one could approximate the effect of room modes given particular source and listener positions by generating paths leaving the source in the directions $(n_x L_x, n_y L_y, n_z L_z)$ for n_x, n_y, n_z small, and for each path that comes sufficiently close to the listener calculate the corresponding frequencies and amplification or damping factors. The approximate frequency response (with respect to room modes alone) is given by the obtained values, considering frequencies in between to have value 1 (no amplification or damping due to modal coupling).

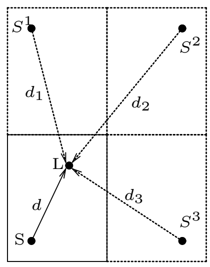
2.2 Comb Filtering

The term comb filtering is applied to a particular type of source-boundary interference corresponding to the superposition of the direct sound and the first reflection on a nearby wall. Letting d be the distance of the source to the wall, the frequencies $f = \frac{kv}{4d}$ for $k = 1, 2, \ldots$ would display a pattern of amplification (with factor $1+\varrho(f)$) and damping (with factor $1 - \varrho(f)$) due to the first reflection. Amplification points occur on the reflection point on the wall and every $\frac{v}{2f}$ (units of length) on the line joining the source and the reflection point; damping is observed at the midpoints on the same line.

Also the distance from the listener to a nearby wall would impinge on the listener a comb filtering effect due to the interference of the direct sound and the sound reflected on the wall behind his/her head. Patterns of amplification and damping would be the same as in the source-wall case.

2.3 Source-Boundary Interference and Ray Tracing

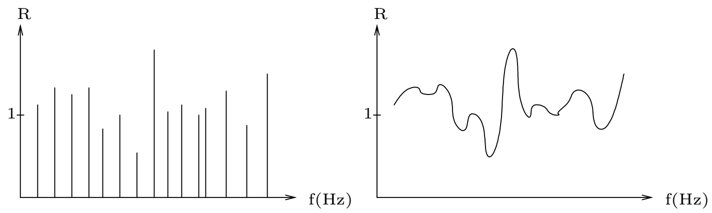
More generally speaking, it is relatively easy to consider all early reflections (up to a certain order) that reach the listener from a particular sound source. This can be achieved by considering virtual images of the sound source in "virtual adjacent rooms", and tracing the direct path from the virtual sound source to the listener, multiplying the response factor by the reflection coefficient ϱ of each (virtual) wall this path traverses for a given frequency [D'Antonio & Cox 1997, Rindel 2000]. If the (direct) distance from the sound source to the listener is d and the distance from the virtual sound source i to the listener is d_i, the response (amplification/damping) factor due to phase difference is $r_i = \varrho^n \cos\left((d_i - d)\frac{2\pi f}{v}\right)$ where n is the number of traversed walls; by considering a set I of virtual sound sources, the corresponding response factor would be $r = 1 + \sum_{i \in I} r_i$.



This approach is closely related to the ray tracing method [Schroeder 1978, D'Antonio & Cox 1997, Rindel 2000], with the difference that by using virtual images no ray is actually generated that never reaches the listener. It should be noticed that this "virtual ray tracing" technique is able to cope with modal coupling and comb filtering effects as well. Nevertheless, since these effects might be perceived over a range of listening positions whereas the ray-tracing takes a single listening position into account, computer power can be spared in treating modal coupling and comb filtering effects separately.

2.4 Combining results

Once approximate frequency response factors have been calculated for each isolate phenomenon (modal coupling, comb filtering and other source-boundary interference), a natural way to combine these results is to multiply response factors for the same frequency. This is to be made over a finite set of frequencies including those involved in modal coupling and comb filtering, as well as selected frequencies in between. Having obtained the values for these isolate frequencies, one could easily calculate a polynomial interpolation to obtain an estimation of the frequency response.



3 Multi-source/listener Optimization Models

Let $r(s,l,d)$ be the approximate response factor as function of the source (s) and listener (l) positions and a description (d) of the room (comprising room dimensions and reflection coefficients). This function might be represented by a vector indexed by a finite set of frequencies $\mathcal{F}$.

An error function is needed to measure deviation from flatness. Let $\bar{r}(s,l,d)$ be the least-squares approximation of $r(s,l,d)$, i.e. the result of the linear regression problem applied to the values of the vector $r(s,l,d)$. Let $e(s,l,d)$ be an error measurement function for the deviation of the response $r(s,l,d)$ from the affine response $\bar{r}(s,l,d)$. Natural choices for this function are the l_1-error $e_1(s,l,d) = \sum_{f\in\mathcal{F}} |r_f(s,l,d) - \bar{r}_f(s,l,d)|$, the quadratic residue $e_2(s,l,d) = \sum_{f\in\mathcal{F}}(r_f(s,l,d) - \bar{r}_f(s,l,d))^2$ and the l_∞-error $e_\infty(s,l,d) = \max_{f\in\mathcal{F}} |r_f(s,l,d) - \bar{r}_f(s,l,d)|$.

Suppose the range of possible source positions is described by a set S (stage) and the range of possible listener positions is described by a set A (audience). These might be assumed to be polyhedra for the sake of simplicity. Let D describe the constraints on the description of the room. This description will typically involve box constraints (i.e. maximal and minimal values) on the dimensions, and a finite set of possible values for reflection coefficients (as related to available materials).

One approach for the optimization model is to ask that the frequency response be optimal in the average case, i.e. in an average source-listener pair. This corresponds to

$$\begin{cases} \min & \int_{s\in S}\int_{l\in A} e(s,l,d) \\ \text{subject to} & d \in D. \end{cases}$$

The integrals on the problem above might be computed approximately, i.e. as sums over a finite number of pairs (s,l).

Another possibility is trying to guarantee that the worst possible combination of source and listener positions will have the best possible frequency response with respect to the chosen measure of deviation from flatness. This corresponds to

$$\begin{cases} \min & \sup_{s\in S,\ l\in A} e(s,l,d) \\ \text{subject to} & d \in D. \end{cases}$$

The solution algorithms for these problems will have to tackle the following difficulties: lack of a closed formula for the objective function (it depends on a simulation); no convexity-like properties in the objective function; mixing of continuous and discrete variables leading to a combinatorial behavior. The most likely outcome is the derivation and implementation of a global optimization heuristic method tailored for the structure of the problem, with techniques like those in [Horst & Pardalos 1995].

4 Conclusions and Further Research

This paper describes optimization models for the design of critical listening rooms aiming at a nearly flat frequency response. Various design options and corresponding optimization models have been presented.

Many of the difficulties of computing frequency response without physical measurements have been discussed, and an implementation of an approximate frequency response function based on identifying modal nodes and comb filters and using the virtual ray tracing model has been proposed. Plenty of computational tests and comparison to real values obtained by direct measuring are still required in order to verify the adequacy of the proposed procedure for the purposes of optimal room design.

Next comes the study of the mathematical structure of the functions $r(s, l, d)$ and $e(s, l, d)$ and their relations to the Stage and Audience sets, in order to minimize the computer time involved in the calculations of the objective functions $\int_{s \in S} \int_{l \in A} e(s, l, d)$ and $\sup_{s \in S,\ l \in A} e(s, l, d)$, as well as efficient ways of improving the design variable d.

Finally, the completed implementation is intended to be openly available under a GPL-type public license.

References

Beranek, Leo L. (1962) *Music, Acoustics, and Architecture*, New York: John Wiley and Sons.

Beranek, Leo L. (1993) *Acoustics*, New York: Acoustical Society of America.

D'Antonio, P. & Cox, T.J. (1997) *ROOM OPTIMIZER: A Computer Program to Optimize the Placement of Listener, Loudspeakers, Acoustical Surface Treatment and Room Dimensions in Critical Listening Rooms*, 103rd Convention of the Audio Engineering Society, Preprint 4555, Paper H-6, New York.

Horst, R. & Pardalos, P. M. (1995) *Handbook of Global Optimization*, Kluwer Academic Publishers.

Iazzetta, F. H.; Kon, F. & Silva, F. (2001) *ACMUS: Design and Simulation of Musical Listening Environments*, Proceedings of the 8th Brazilian Symposium on Computer Music, Fortaleza.

Rindel, J. H. (1997) *Computer Simulation Techniques for the Acoustical Design of Rooms - how to treat reflections in sound field simulation*, Proceedings of ASVA'97, pp. 201-208, Tokyo.

Rindel, J. H. (2000) *The Use of Computer Modeling in Room Acoustics*, Journal of Vibroengineering, Vol. 3, No. 4, pp. 41-72.

Schroeder, M. R. (1978) *Music Perception in Concert Halls*, Paper given at a seminar organized by the Committee for the Acoustics of Music, Royal Swedish Academy of Music 26.

Warusfel, O. (1995) *Predictive acoustics software and computer aided optimization in room acoustics*, 15th Intl. Congress on Acoustics, Trondheim, Norway, pp. 693-696.