Projections on Symmetries and Self-similarities

Pedro Henrique de Faria, Jônatas Manzolli

Interdisciplinary Nucleus of Sound Communication (NICS)
Music Department – University of Campinas(UNICAMP)
pedro.bassclef@gmail.com , jotamanzo@hotmail.com

Abstract: *this article presents concepts and methods developed to compose the rhythmic structure of "Projections on Symmetries and Self-similarities", a piece for 9 percussionists. It mainly describes the mathematical modelling used in a compositional platform to compose this piece and also comments further application in a series of musical compositions.*

1. Introduction

Composers such as Lejaren Hiller, Milton Babbitt, Iannis Xenakis and Gottfried Koenig were of huge significance in formal musical composition they were pioneers in developing Computer Assisted Composition (CAC) system. This article presents a compositional process that works in line with the CAC paradigm. Nevertheless, in an innovative way, we use tables to describe evolution of parameters allowing the composer to instantaneously see, analyze and control the structure of a determined excerpt, section or the whole piece. Shortly, it enables the composer to extract and explore the maximum variability of the material. The inspiration for the work reported here was Achorripsis (1957), where Xenakis (1971) worked with clouds and density of sounds controlled by the Poisson distribution (Arsenault, 2002). The use of tables developed in this study is also correlated to Koenig's Tendency Masks applied to PR-1 and PR-2 (Laske, 1981).

2. Concepts and Architecture

In this section will be presented the implementation of the structural concept of the piece in two main aspects: a) Section generation: from the total number of measures given freely by the composer, the computer uses the Golden Mean to calculate the measures for each section of the piece. This sequence of values in conjunction to other parameters (i.e. densities, variation of densities and number of events per layer) correlates macro-similarities and symmetries. b) Densities generation: this parameter inputs to the computer composer's wish of complexity described by the subdivisions of a main tempo unity. The composer feeds the system with a desired complexity value that is taken as the parameter λ of the Poisson distribution and the computer outputs values describing tendencies of rhythm subdivisions.

The project of the composition presented here is based on the concept of similarity among rhythmic patterns and structural levels. An algorithm generates rhythmic material controlling subdivisions of a main duration unity. Complementary, this work also looks for constructing symmetries throughout layers and sections of the piece, where the control of density takes an important role as a macroscopic unity builder. The formal structural division of the sections follows the Golden Mean that in conjunction with sections and density controls provides balancing and favours the fluidity of the music. Second important aspect is the planning of the percussion instrumentation. It is structured on 5 tom-toms (from extremely low to extremely high), 3 woodblocks (also organized in the same way) and a Suspended Cymbal. The instrumentation requires 3 types of mallets for cymbal and tom-toms: Hard, Medium and Soft (very clear attack and minimum resonance; identifiable attack with a minimum resonance; without a clear attack and maximum resonance, respectively). The specific material and other features are of performers choice.

2.1. Sections Generation

Once the composer inputs the maximal number of measures, the *section generator* is used to output the number of measures in each section that is expressed by an integer approximation of the Golden mean. We named each section as S_j , the sequence of sections is given by the set $T=\{S_1, S_2, \ldots, S_n\}$ and finally the generation of sections is defined by the following expression:

$$S_{j-1} = INT\left(\frac{S_j}{G} + 0.5\right) \qquad \textbf{(1)}$$

where G=1.61803 is the Golden Mean, the INT() operation outputs an integer number, $j=n, (n-1), (n-2), \ldots, 1$ and the sequence of S_j is stopped when $S_1=1$.

In this piece the total number of measures is $S_n=120$ and the set sections generated by Eq. 1 is:

$$T=\{1,2,4,7,11,17,28,46,74,120\}$$

2.2 Sections densities

Starting upon the initial density parameter λ_1 the sequence of sections densities is generated as follows:

$$P(\lambda_1)=D_1, P(\lambda_2)=D_2, \ldots, P(\lambda_n)=D_n \qquad \textbf{(2)}$$

where $P(\lambda_j)$ is j-nth parameter applied to the Poisson distribution.

The variation of sections densities is correlated to the measures by the following rule[1]:

$$|P(\lambda_{j+1})-P(\lambda_j)| \le |j_{j-1}-j_j| \qquad \textbf{(3)}$$

After processing Poisson distribution we have the corresponding values of *Complexity* that are associated to the *Complexity slider* (controller described in section 3). Thus the higher is λ_j , the higher is the tendency of subdivisions. Consequently more events per measure will be generated. It is important to remark that D=0 leads no chance of subdivision in any instance of the rhythm generation, then we assume that D=1 is the lower possible density.

Therefore, between each section and each measure inside every section, we allow only a specific variation of probability density. Applying the Eq. 3 we generated a crescent sequence of numbers $R=\{1,2,3,4,6,11,18,28,46\}$.

The rules expressed above are applied to generate the density probability of two sets of main instruments: Wood-blocks and Tom-toms. The values are given in a pair $(\lambda_1^t, \lambda_1^w)$ **,** where *t* is the index of Tom-tons and *w* is the Wood-block. Therefore, the composer must input this initial pair (i.e. one for each main instrument set) in the same range for the data processing works properly.

3. Rhythmic Tree

This section presents the procedures used to construct and generate the rhythmic tree that

1This equation is named as triangle inequality and it has its origin in Euclidean geometry and refers to the theorem which states that a triangle, the length of one side is always less than the sum of the lengths of two sides.

controls the final rhythmic subdivision of the piece. These procedures were implemented in the Pure Data platform and they can be used in other pieces of music, including real time applications.

3.1. Rhythmic Tree Structure

A *Rhythm generator* was programmed to generate all rhythmic cells, and it was based on the idea of a linearly numbered probability tree.

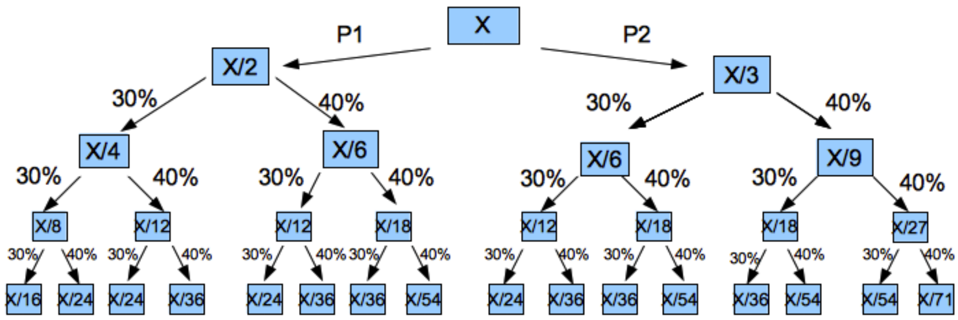

Figure 1 – Probability tree of the *Rhythm generator* and the respective probabilities

Where *x* is the musical figure that corresponds the main tempo unity of the piece (quarter note, eighth, sixteenth etc), and that not necessarily represents one beat. In this piece it was assumed the quarter note as the main tempo unity of the piece. The fractions of the probability tree are converted to the corresponding musical subdivision by hand. The *Rhythm generator* determines which ramification to go based in the scheme probability of a Three valued Markov Chain. The arcs of the Tree have the first level values given by the *Complexity slider,* described in section 3.2. The slider assigns values (P_1 and P_2) to the first level of the binary tree, which decides primarily if the main tempo unity is subdivided by 2, 3 or remain the same. P_1 is the probability of been divided by 2 and P_2 by 3 and P_0, always with fixed value of 30%, is the chance of the figure remain indivisible. As the *Complexity slider* sends values from 0 to 100, this total is normalized to maximum of 70%, thus the $P_0+P_1+P_2 = 100$. After decide for the set $\{P_0,P_1,P_2\}$ the following tree arcs keeps the same probability values (as shown in figure 1).

3.2 Generation

The generated material tends to be very homogenous throughout the piece as probabilities are fixed. This provides some characteristics to the whole piece such as unity and symmetric development in many aspects (textures, expansions and fragmentations of cells). Another notable characteristic of the generated material is that the music is always placed in blocks. Observing these prior points on the matter of generation, graphics were created to allow the composer to follow the progression of the last 15 generations (*x* axis) of densities and their values can be printed on the console for further analysis. This helps the composer predicts the curve tendency of events in each measure/layer. Their range in *y* axis is from 0 to 100 (following the *Complexity slider* range).

Figure 2 – Transcription example of the subdivisions outputted by *Rhythm generator* into score.

4. Discussion and Conclusion

This compositional process and platform operates in a particular way. It approaches a hybrid correlation between deterministic and probabilistic compositional modelling. The fixed architecture of the macro-structure (i.e. the number of total measures, their respective sections and variation of densities) is contrasted with an important degree of liberty. This conducts the composer to a well-shaped and balanced united form and at the same time provides the possibility of composing a wide variety of pieces using the same computer platform by changing a few or even none parameters. A further development could be an implementation of a Probability Tree with no fixed values and this might lead to a generation of a highly heterogeneous material.

Finally, it is possible to observe a more intricate relation between this proposition and Xenakis and Koenig works: a) *about the correlation to percussion music of Xenakis* (*Achorripsis, Pleiades):* in the treatment of densities and textures. Material conceived in blocks and the role of probability distributions (Poisson for instance) in designing these overall densities and textures. Xenakis as well had some personal influence after the main generated data, before the transcription of the final score; b) *about the composer's liberty over the material organization* reminds at the same time Koenig PR-1 and PR-2, where "clearly, the composer using PR-1 is largely a finder of musical sense embodied by computed results. By contrast, the user of PR-2 is a designer who defines both the database and the procedures brought to bear on it. Once computation is complete, the composer using PR-2 has only the task of redefining the input data should they have failed to produce the intended result" as described by Laske (1981). Here, the composer have flexibility to choose in which level of construction he will work the most. This provides variety on the possibilities of detailing some discourse or articulation of material, as selecting and exchanging an amount of outputted data through sections or excerpts, for example.

5. References

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